Accurate measurement of the $D^0 - \bar{D}^0$ mixing parameters

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We propose a new method to determine the mass and width differences of the two D meson mass-eigenstates as well as the CP violating parameters associated with $D^0 - \bar{D}^0$ mixing. We show that an accurate measurement of all the mixing parameters is possible for an arbitrary CP violating phase, by combining observables from a time dependent study of D decays to a doubly Cabibbo suppressed mode with information from a CP eigenstate. As an example we consider $D^0 \to K^{*0}\pi^0$ decays where the K^{*0} is reconstructed in both $K^+\pi^-$ and $K_S\pi^0$. We also show that decays to the CP eigenstate $D \to K^+K^-$ together with $D \to K^+\pi^-$ decays can be used to extract all the mixing parameters. A combined analysis using $D^0 \to K^{*0}\pi^0$ and $D \to K^+K^-$ can also be used to reduce the ambiguity in the determination of parameters.

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Evidence for mixing in the neutral D meson system has recently been reported [1, 2, 3] by the Belle and BaBar collaborations. These experiments find non-vanishing width and mass differences between the two neutral D mass eigenstates assuming negligible CP violation. In this letter we propose a method to determine all the mixing parameters accurately allowing for arbitrary CP violation.

Within the Standard Model CP violation in the D system is negligible. Hence observation of CP violation would be a good signal for New Physics (NP) [4]. While no CP violation has been seen in $D - \bar{D}$ mixing [3], with the current precision large possible NP contributions are not ruled out.

We show that using the doubly Cabibbo suppressed (DCS) mode $D \to K^{*0}\pi^0$ and its conjugate modes, we can solve for all the $D - \bar{D}$ mixing parameters. This is possible if the $K^{*0}/\bar{K}^{*0}$ is reconstructed both in the self tagging $K^\pm\pi^\mp$ mode and in the CP eigenstate $K_S\pi^0$ mode. While the CP eigenstates $D \to K^+K^-$ cannot alone be used to determine all the mixing parameters, we demonstrate that minimal additional information from DCS modes allows determination of all parameters. This approach may provide the optimal method to determine all the parameters with current data. In both these cases, the parameters can be determined accurately even in the limit of a small or vanishing CP violating mixing phase ϕ. It has recently been proposed to use the singly Cabibbo suppressed (SCS) $D \to K^*K$ modes to determine the mixing parameters [5, 6]. However, if ϕ is zero, these methods would be feasible only if the strong phase involved is measured elsewhere. The strong phase can be measured using a Dalitz plot analysis [7]. In the absence of the strong phase information, these modes cannot be used to determine the mixing parameters accurately when ϕ is small, since they can only be expressed as ratio of small quantities.

Our study of the various modes allows us to conclude that in the limit of small ϕ, an accurate measurement of all mixing parameters is possible only if the method also allows the determination of the parameters in the case $\phi = 0$. While mixing parameters can be determined using decays to SCS non-CP eigenstates alone, an accurate measurement of mixing parameters in the limit of small ϕ is possible, only by adding information from decays to CP-eigenstates or if the strong phase is measured independently elsewhere [5, 6, 7, 8, 9, 10]. The $D \to K^{*0}\pi^0$ modes are an example where it is possible to measure all the mixing parameters and the strong phase using only related final states, thereby reducing systematic errors. The methods discussed in this letter do not have systematic errors associated with the parameterization of the resonant content of the Dalitz plot [3] and hence are model-independent.

The neutral D mass eigenstates are related to the weak eigenstates by, $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$. The mass and width differences of these eigenstates are popularly written [11] in terms of the dimensionless variables,

$$x \equiv \frac{\Delta M}{\Gamma} = \frac{M_1 - M_2}{\Gamma} \quad \text{and} \quad y \equiv \frac{\Delta\Gamma}{2\,\Gamma} = \frac{\Gamma_1 - \Gamma_2}{2\,\Gamma},$$

where Γ is the average of the widths of the two mass eigenstates. If the magnitude of q/p differs from unity and/or the weak phase $\phi = \arg(q/p)$ is nonvanishing, this would signal CP violation. We consider mixing to be the only source of CP violation and assume that the decay amplitudes themselves have no weak phase [12].

In the limit $x \ll 1$, $y \ll 1$ and $\Gamma t \ll 1$, the time dependent decay rates for a D^0 decaying to a final state f and $D^0 \to f$ and $\bar{D}^0 \to \bar{f}$ have the form:

$$|A(D^0(t) \to f)|^2 = e^{-\Gamma t}\Big[X_f + Y_f\Gamma t + Z_f(\Gamma t)^2 + \cdots\Big] \quad (1)$$

$$|A(\bar{D}^0(t) \to \bar{f})|^2 = e^{-\Gamma t}\Big[\bar{X}_f + \bar{Y}_f\Gamma t + \bar{Z}_f(\Gamma t)^2 + \cdots\Big]. \quad (2)$$

We first consider the DCS mode $D^0 \to K^{*0}\pi^0$ and its conjugate mode $\bar{D}^0 \to \bar{K}^{*0}\pi^0$, with the $K^{*0}/\bar{K}^{*0}$ reconstructed in the self-tagging $K^{\pm}\pi^{\mp}$ modes. The coefficient functions of the constant, linear and quadratic terms in $(\Gamma\, t)$ in the time dependent decay rates are given by,

$$\begin{aligned} X_{K^*_\pi} &= \bar{X}_{K^*_\pi} = \left|A_{K^*_\pi}\right|^2 r^2_{K^*_\pi}, && (3)\\ Y_{K^*_\pi} &= \left|\frac{q}{p}\right|\left|A_{K^*_\pi}\right|^2 r_{K^*_\pi}\big(y'_{K^*_\pi}\cos\phi - x'_{K^*_\pi}\sin\phi\big), && (4)\\ \bar{Y}_{K^*_\pi} &= \left|\frac{p}{q}\right|\left|A_{K^*_\pi}\right|^2 r_{K^*_\pi}\big(y'_{K^*_\pi}\cos\phi + x'_{K^*_\pi}\sin\phi\big), && (5)\\ Z_{K^*_\pi} &= \left|\frac{q}{p}\right|^2\left|A_{K^*_\pi}\right|^2 \frac{x^2+y^2}{4} \qquad \text{and} && (6)\\ \bar{Z}_{K^*_\pi} &= \left|\frac{p}{q}\right|^2\left|A_{K^*_\pi}\right|^2 \frac{x^2+y^2}{4}, && (7) \end{aligned}$$

where,

$$\begin{aligned} x'_{K^*_\pi} &= (x\cos\delta_{K^*_\pi} + y\sin\delta_{K^*_\pi}), && (8)\\ y'_{K^*_\pi} &= (y\cos\delta_{K^*_\pi} - x\sin\delta_{K^*_\pi}), && (9) \end{aligned}$$

with $A_{K^*_\pi} \equiv A(D^0 \to \bar{K}^{*0}\pi^0)$ and the ratio of the DCS to CF amplitude defined as

$$-r_{K^*_\pi} e^{-i\delta_{K^*_\pi}} \equiv \frac{A(D^0 \to K^{*0}\pi^0)}{A(D^0 \to \bar{K}^{*0}\pi^0)} = \frac{A(D^0 \to K^{*0}\pi^0)}{A(\bar{D}^0 \to K^{*0}\pi^0)}.$$

The amplitude $\left|A_{K^*_\pi}\right|$ can easily be measured using the time integrated rate for the Cabibbo favored (CF) mode $D^0 \to \bar{K}^{*0}\pi^0$ which is given by,

$$\int_0^\infty |A(D^0(t) \to \bar{K}^{*0}\pi^0)|^2 dt \;\approx\; \left|A_{K^*_\pi}\right|^2, \qquad (10)$$

where terms of the order of x^2 or y^2 and $r_{K^*_\pi}\, x$ or $r_{K^*_\pi}\, y$ are neglected compared to unity, as these are expected to be $\mathcal{O}(10^{-4})$ or less. The ratio $r_{K^*_\pi}$ can be determined using Eqs. (3) and (10). The observables $Z_{K^*_\pi}$ and $\bar{Z}_{K^*_\pi}$ also readily determine $|q/p|$ and x^2+y^2 to be:

$$\begin{aligned} \left|\frac{q}{p}\right|^4 &= \frac{Z_{K^*_\pi}}{\bar{Z}_{K^*_\pi}}, && (11)\\ f^2 \equiv x^2+y^2 &= 4\frac{\sqrt{Z_{K^*_\pi}\bar{Z}_{K^*_\pi}}}{\left|A_{K^*_\pi}\right|^2}. && (12) \end{aligned}$$

The two linear terms in the time dependent DCS decay rates $Y_{K^*_\pi}$ and $\bar{Y}_{K^*_\pi}$ may be re-expressed in terms of two more convenient observables $Y^{(+)}_{K^*_\pi}$ and $Y^{(-)}_{K^*_\pi}$ as follows

$$\begin{aligned} Y^{(+)}_{K^*_\pi} &= \frac{\bar{Y}_{K^*_\pi}\,|q|^2 + Y_{K^*_\pi}\,|p|^2}{2\, r_{K^*_\pi}\left|A_{K^*_\pi}\right|^2 |q|\,|p|} = y'_{K^*_\pi}\cos\phi, && (13)\\ Y^{(-)}_{K^*_\pi} &= \frac{\bar{Y}_{K^*_\pi}\,|q|^2 - Y_{K^*_\pi}\,|p|^2}{2\, r_{K^*_\pi}\left|A_{K^*_\pi}\right|^2 |q|\,|p|} = x'_{K^*_\pi}\sin\phi. && (14) \end{aligned}$$

Note that the observable $Y^{(-)}_{K^*_\pi}$ may be difficult to measure in the small ϕ limit.

The $K^{*0}/\bar{K}^{*0}$ in the final state could also have been reconstructed in the neutral $K_S\pi^0$ mode, resulting in an additional observable. A unique feature of the final state $K_S\pi^0\pi^0$ is that it includes contributions from both $K^{*0}\pi^0$ as well as $\bar{K}^{*0}\pi^0$ states; the amplitude for this final state is thus a sum of the CF and DCS amplitudes,

$$\begin{aligned} \left|A_{K_S\pi\pi}\right|^2 &\equiv |A(D^0 \to K_S\pi^0\pi^0)|^2\\ &= |A_{K^*_\pi}|^2(1 + r^2_{K^*_\pi} - 2\, r_{K^*_\pi}\cos\delta_{K^*_\pi}). && (15) \end{aligned}$$

Since the decay mode involves two neutral pions it will not be easy to perform a time dependent study. Hence, we consider only the time integrated decay rate for this mode. The amplitudes $A(D^0 \to K_S\pi^0\pi^0)$ and $A(\bar{D}^0 \to K_S\pi^0\pi^0)$ are equal since $K_S\pi^0\pi^0$ is a CP eigenstate. Hence, the time integrated decay rate for $D^0 \to K_S\pi^0\pi^0$ is given by:

$$\begin{aligned} &\int_0^\infty |A(D^0(t) \to K_S\pi^0\pi^0)|^2 dt\\ &\approx \left|A_{K_S\pi\pi}\right|^2\Big[1 + \frac{q}{p}(y\,\cos\phi - x\,\sin\phi)\Big]\\ &\approx |A_{K^*_\pi}|^2\Big[1 + \frac{q}{p}(y\,\cos\phi - x\,\sin\phi) - 2r_{K^*_\pi}\cos\delta_{K^*_\pi}\Big], && (16) \end{aligned}$$

where, terms of order x^2, y^2 and $r_{K^*_\pi}\, x$, $r_{K^*_\pi}\, y$ as well as $r^2_{K^*_\pi}$ are once again neglected compared to unity.

Using Eqs. (13) and (14), one obtains the following solutions for $\tan^2\phi$ and $y'^2_{K^*_\pi}$:

$$\begin{aligned} \tan^2\phi &= \frac{2f^2 - \mathcal{F}_{K^*_\pi} - \sqrt{\mathcal{F}_{K^*_\pi}{}^2 - 4f^2 Y^{(+)}_{K^*_\pi}{}^2}}{\mathcal{F}_{K^*_\pi} + \sqrt{\mathcal{F}_{K^*_\pi}{}^2 - 4f^2 Y^{(+)}_{K^*_\pi}{}^2}} && (17)\\ y'^2_{K^*_\pi} &= \frac{\mathcal{F}_{K^*_\pi} - \sqrt{\mathcal{F}_{K^*_\pi}{}^2 - 4f^2 Y^{(+)}_{K^*_\pi}{}^2}}{2} && (18) \end{aligned}$$

where, $\mathcal{F}_{K^*_\pi} = f^2 - Y^{(-)}_{K^*_\pi}{}^2 + Y^{(+)}_{K^*_\pi}{}^2$. The ambiguity in the solutions of the quadratic equations in $\tan^2\phi$ and $y'^2_{K^*_\pi}$ is fixed by the correct limiting solution in the $\phi = 0$ limit. Further, expressing $\cos\delta_{K^*_\pi}$ in terms of x, y and $x'_{K^*_\pi}$, $y'_{K^*_\pi}$, Eq. (16) may be rewritten as a quadratic equation in x/y,

$$\left[B^2 - \zeta^2\right]\left(\frac{x}{y}\right)^2 + 2\,A\,B\,\frac{x}{y} + A^2 - \zeta^2 = 0, \qquad (19)$$

where,

$$A = 2r_{K^*_\pi}\, y'_{K^*_\pi} - \left|\frac{q}{p}\right|\frac{Y^{(+)}_{K^*_\pi} f^2}{y'_{K^*_\pi}}, \quad B = 2\, r_{K^*_\pi}\, x'_{K^*_\pi} + \left|\frac{q}{p}\right|\frac{Y^{(+)}_{K^*_\pi} f^2}{x'_{K^*_\pi}},$$

$$\zeta = \left(\frac{Br(D^0 \to K_S\pi^0\pi^0)}{Br(D^0 \to \bar{K}^{*0}\pi^0)} - 1\right) f, \qquad (20)$$

allowing x/y to be solved with a four-fold ambiguity. x and y can thus be individually determined using Eq. (12). The solution obtained is finite even if $\phi = 0$, with a correction term of order $Y^{(-)}_{K^*_\pi}$. Hence an accurate estimation

is possible even if ϕ is tiny. We show below that the ambiguity in x/y can be reduced if information from K^+K^- modes is added as well.

We next consider the time dependent decay of a D meson to a singly Cabibbo suppressed (SCS) CP eigenstate such as $D \to K^+K^-$ or $D \to \pi^+\pi^-$. To be specific, we will consider only the K^+K^- final state, but the conclusions can be straightforwardly applied to any other SCS-CP eigenstate. For this final state, the strong phase is identically zero; and hence, the coefficients of the constant and linear terms in $(\Gamma\, t)$, defined using the time dependent decay in Eq.(1) reduce to the simple form:

$$X_{KK} = \bar{X}_{KK} = \left|A_{KK}\right|^2 \quad (21)$$

$$Y_{KK} = -|\frac{q}{p}|\left|A_{KK}\right|^2(-x\sin\phi + y\cos\phi), \quad (22)$$

$$\bar{Y}_{KK} = -|\frac{p}{q}|\left|A_{KK}\right|^2(x\sin\phi + y\cos\phi) \quad (23)$$

Unlike the DCS modes where the term quadratic in $\Gamma\, t$ is enhanced by the ratio of CF to DCS rates, in the SCS modes all time dependent terms are of the same order in $\sin\theta_c$, hence quadratic and higher terms in $\Gamma\, t$ cannot be extracted. Assuming $|q/p| \approx 1$ and $\phi = 0$, the linear term in $\Gamma\, t$ can directly measure y as has been done in Ref. [1]. However, the time dependent study of only the SCS CP eigenstates does not allow x to be determined, even in the limit $|q/p| \approx 1$ and $\phi = 0$.

We will show that if we also include in this analysis the quadratic terms in $(\Gamma\, t)$ from the time dependent decay rates of DCS modes such as $K\pi$, all the mixing parameters can be solved without approximation. For $D^0 \to K^+\pi^-$ and $\bar{D}^0 \to K^-\pi^+$, the coefficient functions of the quadratic terms in $(\Gamma\, t)$ will be analogous to those for the $K^*\pi$ mode given in Eq. (6). Hence, the corresponding observables $Z_{K\pi}$ and $\bar{Z}_{K\pi}$ readily determine $|q/p|$ and $f^2 = x^2 + y^2$. Alternatively, $|q/p|$ and f^2 could be measured using time integrated wrong sign relative to right sign semileptonic decay rates. Having obtained $|q/p|$ and f^2, ϕ and x/y can easily be determined from $D \to K^+K^-$. Using Eqs. (21) – (23), which can be re-expressed as,

$$Y_{KK}^{(+)} = \frac{\bar{Y}_{KK}\,|q|^2 + Y_{KK}\,|p|^2}{2\,X_{K\pi}|q|\,|p|} = -y\cos\phi, \quad (24)$$

$$Y_{KK}^{(-)} = \frac{\bar{Y}_{KK}\,|q|^2 - Y_{KK}\,|p|^2}{2\,X_{K\pi}|q|\,|p|} = -x\sin\phi, \quad (25)$$

solution for x^2/y^2 and ϕ can be straightforwardly written

$$\frac{x^2}{y^2} = \frac{\mathcal{F}_{KK} - 2{Y_{KK}^{(+)}}^2 + \sqrt{\mathcal{F}_{KK}^2 - 4\,f^2\,{Y_{KK}^{(+)}}^2}}{2{Y_{KK}^{(+)}}^2},$$

$$\tan^2\phi = \frac{2f^2 - \mathcal{F}_{KK} - \sqrt{\mathcal{F}_{KK}^2 - 4\,f^2\,{Y_{KK}^{(+)}}^2}}{\mathcal{F}_{KK} + \sqrt{\mathcal{F}_{KK}^2 - 4\,f^2\,{Y_{KK}^{(+)}}^2}},$$

where $\mathcal{F}_{KK} = f^2 + {Y_{KK}^{(+)}}^2 - {Y_{KK}^{(-)}}^2$. We once again examine in detail the solution for the case of small ϕ. If ϕ is small, the measured value of $Y_{KK}^{(-)}$ will be small. The above solutions can then be written as a series in ${Y_{KK}^{(-)}}^2$:

$$\frac{x^2}{y^2} = \frac{f^2 - {Y_{KK}^{(+)}}^2}{{Y_{KK}^{(+)}}^2} - \frac{{Y_{KK}^{(-)}}^2\,f^2}{{Y_{KK}^{(+)}}^2(f^2 - {Y_{KK}^{(+)}}^2)} + \mathcal{O}({Y_{KK}^{(-)}}^4)$$

$$\tan^2\phi = \frac{{Y_{KK}^{(-)}}^2}{f^2 - {Y_{KK}^{(+)}}^2} + \mathcal{O}({Y_{KK}^{(-)}}^4),$$

and therefore x^2/y^2 is finite even for small ϕ.

As mentioned earlier, if information from K^+K^- modes is added to that from the $K^*\pi$ modes, a reduction in ambiguity is possible. If the observables $Y_{KK}^{(+)}$ and $Y_{KK}^{(-)}$ are used, then $\cos\delta_{K^*\pi}$ can be obtained purely in terms of observables directly from Eq. (16). Knowing $\cos\delta_{K^*\pi}$, Eqs. (13), (14) and (24), (25) can be used to get,

$$\frac{x^2}{y^2} = \frac{\left(Y_{K^*\pi}^{(+)} - \cos\delta_{K^*\pi}Y_{KK}^{(+)}\right)^2}{{Y_{KK}^{(+)}}^2(1 - \cos^2\delta_{K^*\pi})}.$$

Combining this with the sum $x^2 + y^2$, x^2 and y^2 can be individually determined. Further, Eqs.(25) and (14) can be used to obtain,

$$\frac{y}{x} = \frac{-1}{\sin\delta_{K^*\pi}}\left(\frac{Y_{K^*\pi}^{(-)}}{Y_{KK}^{(-)}} + \cos\delta_{K^*\pi}\right), \quad (26)$$

which helps in reducing the ambiguities in x and y from four-fold to two-fold.

Recently a method was proposed [5] to determine all the mixing parameters using the $D \to K^*K$ modes. This mode is singly Cabibbo suppressed (SCS) but unlike the K^+K^- mode it is not a CP eigenstate. Hence one can study time dependence in all the four modes: $D^0 \to K^{*\pm}K^{\mp}$ and $\bar{D}^0 \to K^{*\mp}K^{\pm}$. The coefficients of the constant and linear terms in $(\Gamma\, t)$ may be written as:

$$\begin{aligned}
Y_{K^{*+}K^-} &= |\frac{q}{p}|\left|A_{K^*K}\right|^2 r_{K^*K}\left(y'_{K^{*+}K^-}\cos\phi - x'_{K^{*+}K^-}\sin\phi\right)\\
\bar{Y}_{K^{*+}K^-} &= |\frac{p}{q}|\left|A_{K^*K}\right|^2 r_{K^*K}\left(y'_{K^{*+}K^-}\cos\phi + x'_{K^{*+}K^-}\sin\phi\right)\\
Y_{K^{*-}K^+} &= |\frac{q}{p}|\left|A_{K^*K}\right|^2 r_{K^*K}\left(y'_{K^{*-}K^+}\cos\phi - x'_{K^{*-}K^+}\sin\phi\right)\\
\bar{Y}_{K^{*-}K^+} &= |\frac{p}{q}|\left|A_{K^*K}\right|^2 r_{K^*K}\left(y'_{K^{*-}K^+}\cos\phi + x'_{K^{*-}K^+}\sin\phi\right)\\
X_{K^{*+}K^-} &= \bar{X}_{K^{*+}K^-} = \left|A_{K^*K}\right|^2\\
X_{K^{*-}K^+} &= \bar{X}_{K^{*-}K^+} = \left|A_{K^*K}\right|^2 r_{K^*K}^2
\end{aligned} \quad (27)$$

where, $A_{K^*K} = A(D^0 \to K^{*+}K^-) = A(\bar{D}^0 \to K^{*-}K^+)$ and r_{K^*K} is defined as

$$-r_{K^*K}\,e^{i\delta_{K^*K}} = \frac{A(\bar{D}^0 \to K^{*+}K^-)}{A(D^0 \to K^{*+}K^-)} = \frac{A(D^0 \to K^{*-}K^+)}{A(\bar{D}^0 \to K^{*-}K^+)}.$$

It may also be noted that $y'_{K^{*-}K^+}$ and $x'_{K^{*-}K^+}$ are different from $y'_{K^{*+}K^-}$ and $x'_{K^{*+}K^-}$ and are defined as:

$$\begin{aligned} x'_{K^{*+}K^-,K^{*-}K^+} &= (x\cos\delta_{K^*K} \pm y\sin\delta_{K^*K}), \\ y'_{K^{*+}K^-,K^{*-}K^+} &= (y\cos\delta_{K^*K} \mp x\sin\delta_{K^*K}). \end{aligned} \tag{28}$$

One may conclude that the six observables in Eqs. (27) can be used to evaluate the six parameters $\left|A_{K^*K}\right|^2$, $r^2_{K^*K}$, x, y, ϕ and δ_{K^*K} assuming the value of $|q/p|$ from elsewhere. However, note that if the mixing phase $\phi = 0$, then, the number of observables reduces to four (since now, $|q|^2\bar{Y}_{K^{*+}K^-} = |p|^2 Y_{K^{*+}K^-}$ and $|q|^2\bar{Y}_{K^{*-}K^+} = |p|^2 Y_{K^{*-}K^+}$) and a solution of all the five parameters is not possible without some additional information. Moreover, for small but nonvanishing ϕ the solution for the ratio x^2/y^2 will be inaccurate as it will depend on ratio of two very small observables. To see this, let us define, $Y^{(+)}_{K^{*+}K^-} = y'_{K^{*+}K^-}\cos\phi$, $Y^{(-)}_{K^{*+}K^-} = x'_{K^{*+}K^-}\sin\phi$, $Y^{(+)}_{K^{*-}K^+} = y'_{K^{*-}K^+}\cos\phi$ and $Y^{(-)}_{K^{*-}K^+} = x'_{K^{*-}K^+}\sin\phi$, which can all be determined in terms of observables using Eqs. (27). It then, it follows that:

$$\frac{x^2}{y^2} = \frac{(Y^{(-)}_{K^{*+}K^-} + Y^{(-)}_{K^{*-}K^+})(Y^{(+)}_{K^{*-}K^+} - Y^{(+)}_{K^{*+}K^-})}{(Y^{(-)}_{K^{*+}K^-} - Y^{(-)}_{K^{*-}K^+})(Y^{(+)}_{K^{*-}K^+} + Y^{(+)}_{K^{*+}K^-})}. \tag{29}$$

It is clear that the RHS involves the ratio of two small quantities when ϕ is small. It is easy to see that this situation is easily alleviated if δ_{K^*K} is measured elsewhere [7]. In fact, the knowledge of δ_{K^*K} not only allows the additional determination of $|q/p|$ [6], but also enables an accurate measurement of mixing parameters.

We now estimate the values of the mixing parameters that can be obtained using the current data for $D \to K^+K^-/\pi^+\pi^-$ [1] and the world average for $x^2 + y^2$ [15]. Assuming $|q/p| = 1$, we obtain $Y^{(+)}_{KK} = 0.0131 \pm 0.0041$, $Y^{(-)}_{KK} = -0.0001 \pm 0.0034$ and $f^2 = 0.00042 \pm 0.00022$, resulting in $|x| = (1.57 \pm 0.56)\times 10^{-2}$, $|y| = (1.31 \pm 0.41)\times 10^{-2}$ and value (up to ambiguities) of $\phi = \pm(0.36 \pm 12.36)^o$. We emphasize that our method allows the determination of mixing parameters even for $|q/p| \neq 1$; the choice $|q/p| = 1$ has been made here, only due to lack of complete tabulated data.

An estimate of the precision to which the mixing parameters can be measured, using the $D \to K^*\pi$ modes, requires the number of reconstructed $D \to K^{*0}\pi^0 \to K^+\pi^-\pi^0$ events. While a branching fraction for this mode has not yet been reported, about 500 events (in $230 fb^{-1}$) for the mode $D^0 \to K^{*+}\pi^- \to K^+\pi^0\pi^-$, have been observed [13]. We present our estimates using two representative values for the ratio of DCS modes:

$$\frac{B(D^0 \to K^{*0}\pi^0 \to K^+\pi^-\pi^0)}{B(D^0 \to K^{*+}\pi^- \to K^+\pi^0\pi^-)} = (0.4, 1.2).$$

These values are chosen to be of the order 0.85, the measured [14] ratio of corresponding CF branching fractions. With an integrated luminosity of $1\,ab^{-1}$ at an e^+e^- B factory we expect about $(4000, 12000)$ $D^0 \to K^{*0}\pi^0 \to K^+\pi^-\pi^0$ events. Interpolating the errors in $D \to K^+\pi^-$ and assuming $\delta = \phi = 0$, the approximate errors on $|x|^2$ and $|y|$ are expected to be $(4.7, 2.7)\times 10^{-4}$ and $(8.9, 5.2)\times 10^{-3}$, respectively.

We have proposed a new method to determine the $D^0 - \bar{D}^0$ mixing parameters x, y, $|q/p|$ and ϕ for arbitrary values of ϕ. The doubly Cabibbo suppressed mode $D^0 \to K^{*0}\pi^0$ reconstructed in two final states ($K^+\pi^-\pi^0$ and $K_S\pi^0\pi^0$) enables the determination of all the mixing parameters. For the $K_S\pi^0\pi^0$ mode, only time integrated measurements are used, while for the $K^+\pi^-\pi^0$ mode time dependent measurements are required. We also show that decays to the CP eigenstate $D \to K^+K^-$ together with $D \to K^+\pi^-$ can be used to extract all the mixing parameters. By combining measurements of $D \to K^{*0}\pi^0$ with results on $D \to K^+K^-$ one can reduce the number of ambiguous solutions for mixing parameters. We estimate that $|x|$, $|y|$ and ϕ can be measured with precision of order 0.6×10^{-2}, 0.4×10^{-2} and 12^o respectively, using data available at present. It should be possible to determine $|x|$, $|y|$ to order 7×10^{-4}, 4×10^{-4} respectively and ϕ to about 1^o at a Super-B factory with an integrated luminosity of $50\,ab^{-1}$ [16].

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